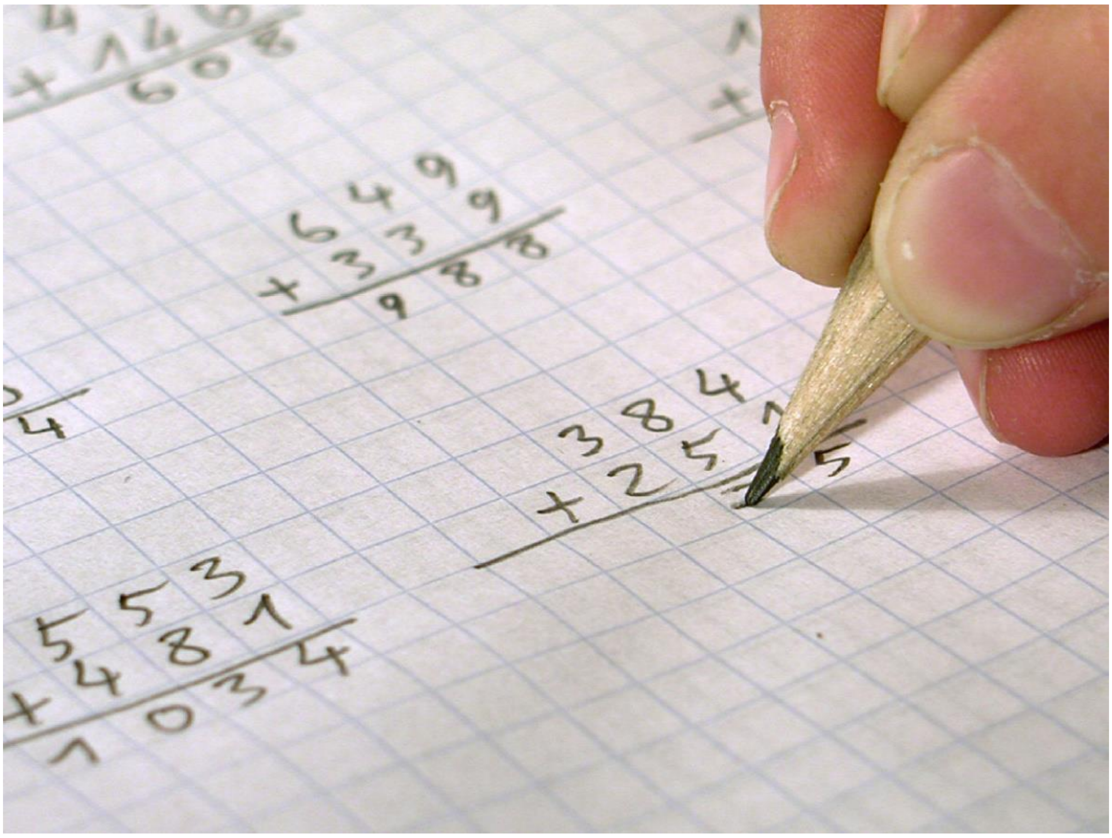
St Anthony's and St Aidan's 6th Form

DEPARTMENT

OF

MATHEMATICS

Introduction to Core Maths



INDUCTION BOOKLET

Student Name:

My teachers are:

INTRODUCTION TO CORE MATHS AT ST ANTHONY'S AND ST AIDAN'S 6TH FORM

Thank you for choosing to study Core Mathematics at St Anthony's and St Aidan's 6th Form. Although there are no external examinations in Year 12 you will sit a Core Maths exam at the end of the academic year. In order for you make the best possible start to the course, we have prepared this booklet.

It is important that you spend some time working through the questions in this booklet in the coming weeks - you will need to have a good knowledge of these topics before you commence your course and these revision exercises will help. You should have met most of the topics before at GCSE. Work through the introduction to each chapter, making sure that you understand the examples. Then try the exercise to ensure you understand the topic thoroughly. The answers are given at the back of the booklet.

We hope that you will use this introduction to give you a good start to your Core Maths work and that it will help you enjoy and benefit from the course more.

Mrs Armstrong
Deputy Subject Leader Mathematics

CONTENTS

SECTION 1: PERCENTAGES

To calculate a percentage we should use a multiplier. The multiplier is found by dividing the required percentage by 100.
For increases or decreases we add/subtract the required percentage to/from 100% at the beginning.

Examples

1) Calculate 26% of 412

$0.26 \times 412 = 107.12$

2) Increase 65 by 7%

$100\% + 7\% = 107\%$
$65 \times 1.07 = 69.55$

3) Decrease 130 by 89%

$100\% - 89\% = 11\%$
$130 \times 0.11 = 14.3$

To calculate a successive percentage change we use these multipliers but raise them to a power to represent the number of successive increases or decreases. This is also known as compound interest/decay.

Examples:

1) John invests £2000 for 5 years in a savings account that pays 2.3% compound interest. How much does he have at the end of the 5 years?

$100\% + 2.3\% = 102.3\%$

$2000 \times 1.023^5 = £2240.83$ (2dp)

2) Amy buys a car for £10000. Each year the car depreciates in value by 12%. How much is the car worth after 4 years?

$100\% - 12\% = 88\%$

$10000 \times 0.88^4 = £5996.95$

EXERCISE A Calculate the following

(a) 80ml decreased by 4% (b) £480 decreased by 13% (c) 143g decreased by 40%

(d) 308 decreased by 1.2% (e) 2250 decreased by 0.5% (f) 9kg decreased by 6.03%

EXERCISE B

Question 1: Paul leaves £4000 in the bank for two years.

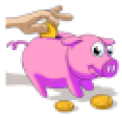
It earns compound interest of 5% per year.
Calculate the total amount Paul has in the bank at the end of the two years.

Question 2: The population of birds on an island is estimated to increase by 10% every year.

The population of birds on the island is 20000.
Calculate an estimate for the population of birds in three years time.

Question 3: The value of a car decreases by 5% each year.

Sophie bought a car two years ago for £10000
Work out the value now.



Question 4: Sam invests £1800 in the bank for four years.

It earns compound interest of 4% each year.
Calculate the total amount Sam has in the bank at the end of four years.

Question 5: A full water tank holds 500 litres.

The tank begins to leak water and is losing 14% of its contents every hour.
Find how much water is left in the tank after 8 hours.

Question 6: The height of a tree increases by 60% each year.

When planted the tree was 40cm tall.
How tall will the tree be in 5 years time.

SECTION 2: ESTIMATION

When estimating we need to round each value to 1 significant figure before doing the calculation.

Example 1: Estimate the value of : $\frac{89+9.6}{14.2-4.9}$

$$\frac{90+10}{10-5}$$

$$\frac{100}{5} = 20$$

Example 2: Estimate the value of: $\frac{74-1.7}{0.47}$

$$\frac{70-2}{0.5}$$

$$\frac{68}{0.5} = 136$$

Exercise A: Estimate the value of the following

(a) $\frac{291+602}{102}$ (b) $\frac{8019}{711-508}$ (c) $\frac{7.14+16.88}{10.96-4.85}$

(d) $\frac{132+291}{31-12}$ (e) $\frac{3890}{9.8\times 51}$ (f) $\frac{42\times 194}{10.3\times 7.8}$

(g) $\frac{18.5\times 51.9}{4.69+20.01}$ (h) $\frac{19.2\times 41.3}{9.9\times 5.1}$

Exercise B: Estimate the value of the following

(a) $\dfrac{52 \times 6.78}{0.51}$	(b) $\dfrac{801 \times 10.04}{0.49}$	(c) $\dfrac{9.8 - 2.9}{0.53}$
(d) $\dfrac{58.46}{6.13 \times 0.505}$	(e) $\dfrac{291 \times 3.95}{0.197}$	(f) $\dfrac{403 \times 3.91}{0.104}$
(g) $\dfrac{7985}{51.28 \times 0.42}$	(h) $\dfrac{304 \times 9.79}{0.602}$	

SECTION 3: AVERAGES AND THE RANGE

The Mean
You will already be familiar with the averages and the range and calculating them from a frequency table.
To calculate the mean from a frequency table we must calculate the fx column then divide its total by the total frequency. When using grouped data we must also add a midpoint column and as we are not using the raw data values the mean from a grouped frequency table will be an estimate.

Example: Calculate the mean of the following data:

Points scored in a game (x)	Frequency	fx
0	9	0
1	11	11
2	18	36
3	7	21
TOTAL	45	68

Mean = $\frac{68}{45} = 1.51$

Example: Calculate an estimate of the mean of the following data:

Points scored in a game (x)	Frequency	Mid point	fx
$15 < x \leq 20$	3	17.5	52.5
$20 < x \leq 25$	6	22.5	135
$25 < x \leq 30$	7	27.5	192.5
$30 < x \leq 40$	4	35	140
TOTAL	20		520

Mean = $\frac{520}{20} = 26$

The Median
To calculate the median from a frequency table we must find the middle value. To find the position of the median we take the total frequency, add 1 and divide by 2. We then count through the frequency using a cumulative frequency column until we get to this value, this then corresponds to the median. When we are working from a grouped frequency table we do not have the raw data values therefore we can only find the class where the median lies.

Example: Calculate the mean of the following data:

Points scored in a game (x)	Frequency	Cumulative frequency
0	9	9
1	11	20
2	18	38 (23rd value is 2)
3	7	
TOTAL	45	

Median = $\frac{45+1}{2} = 23rd\ value = 2$

Example: Calculate an estimate of the mean of the following data:

Points scored in a game (x)	Frequency	Cumulative frequency
$15 < x \leq 20$	3	3
$20 < x \leq 25$	6	9
$25 < x \leq 30$	7	16 (10th and 11th values are both in this group)
$30 < x \leq 40$	4	
TOTAL	20	

Median = $\frac{20+1}{2} = 10.5th\ value = 25 < x \leq 30$

Exercise A:

Calculate the mean:

(a)

Age	Frequency
16	28
17	7
18	3
19	2

(b)

Grade	Frequency
3	16
4	27
5	45
6	49
7	50
8	13

(c)

Siblings	Frequency
0	71
1	25
2	14

(d)

Pocket Money	Frequency
£1	5
£2	34
£3	86
£4	19
£5	3
£6	3

(e)

Star rating	Frequency
0	9
1	12
2	17
3	19
4	21
5	8

(f)

Times visited	Frequency
0	131
1	873
2	599
3	205

Exercise B:

Calculate an estimate of the mean:

(a)

Duration (years)	Frequency
$0 \le d < 10$	9
$10 \le d < 20$	13
$20 \le d < 30$	16
$30 \le d < 40$	2

(b)

Length (cm)	Frequency
$0 \le L < 30$	8
$30 \le L < 60$	43
$60 \le L < 90$	25
$90 \le L < 120$	4

(c)

Mass	Frequency
$20 < m \le 25$	12
$25 < m \le 30$	24
$30 < m \le 35$	17
$35 < m \le 40$	15
$40 < m \le 45$	4

(d)

Height	Frequency
$120 < h \le 130$	51
$130 < h \le 140$	120
$140 < h \le 150$	66
$150 < h \le 160$	59
$160 < h \le 170$	4

Exercise C:

Calculate the median:

(a)

Age	Frequency
18	2
19	3
20	13
21	1

(b)

Shoe Size	Frequency
5	2
6	11
7	5
8	4
9	1

(c)

Number of TVs	Frequency
0	3
1	15
2	9
3	11
4	1

(d)

Days absent	Frequency
0	31
1	8
2	3
3	4
4	1
5	3

(e)

Age	Frequency
5	12
6	20
7	23
8	65

(f)

Goals Scored	Frequency
0	2
1	4
2	5
3	8
4	0
5	1

Exercise D:

Find the group where the median lies:

(a)

Time taken	Frequency
$0 < t \leq 5$	5
$5 < t \leq 10$	14
$10 < t \leq 15$	10
$15 < t \leq 20$	1

(b)

Lifetime (months)	Frequency
$0 < t \leq 12$	1
$12 < t \leq 24$	9
$24 < t \leq 36$	13
$36 < t \leq 48$	56
$48 < t \leq 60$	21

SECTION 4: CO-ORDINATE GEOMETRY

Gradient of a line segment

We can use the following formula to find the gradient of a line segment between 2 points:

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Example 1: Calculate the gradient between points (4,3) and (6, 7)

$$\frac{7-3}{6-4} = \frac{4}{2} = 2$$

Example 2: Calculate the gradient between points (2,10) and (5, 1)

$$\frac{1-10}{5-2} = \frac{-9}{3} = -3$$

Exercise A

Find the gradient between each of the pairs of points

(a) $(1, 4)$ and $(3, 10)$	(b) $(0, 0)$ and $(3, 12)$	(c) $(5, -2)$ and $(9, 14)$
(d) $(-8, 6)$ and $(0, -2)$	(e) $(-5, -9)$ and $(1, 3)$	(f) $(-7, -2)$ and $(1, -4)$
(g) $(-2, 1)$ and $(8, -7)$	(h) $(-2, 9)$ and $(4, 7)$	(i) $(-4.5, 3)$ and $(6, -7.5)$

Equation of a line given 2 points

To find the equation of a straight line given 2 points we must first calculate the gradient as above. We can then use either of the following formulae to calculate the equation of the line:

$$y = mx + c$$

Here we substitute in the gradient and one of the points for x and y and re-arrange to find c. We can then write the equation using our known values of m and c.

$$y - y_1 = m(x - x_1)$$

Here we substitute in the gradient and one of our points for x_1 and y_1 and re-arrange to give us the equation of the line.

The examples below will show you how to use each of these formulae.

Example: Find the equation of the line passing through the points (-2,1) and (3,11)

First we calculate the gradient:

$$\frac{11-1}{3--2} = \frac{10}{5} = 2$$

Using formula 1:

$$y = mx + c$$

$$11 = 2(3) + c$$

$$11 = 6 + c$$

$$c = 5$$

$$y = 2x + 5$$

Using formula 2:

$$y - y_1 = m(x - x_1)$$

$$y - 11 = 2(x - 3)$$

$$y - 11 = 2x - 6$$

$$y = 2x + 5$$

Exercise B

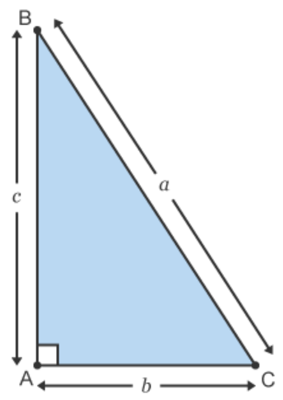
Find the equation of the line passing through these pairs of points:

(a) $(2, 5)$ and $(4, 11)$	(b) $(-4, 2)$ and $(1, 7)$	(c) $(-5, -8)$ and $(-4, -4)$
(d) $(-1, -2)$ and $(-6, 3)$	(e) $(-6, -4)$ and $(-3, 2)$	(f) $(3, 5)$ and $(4, 1)$
(g) $(-5, 4)$ and $(5, 2)$	(h) $(1, 6)$ and $(5, 4)$	(i) $(-10, -5)$ and $(-7, 4)$

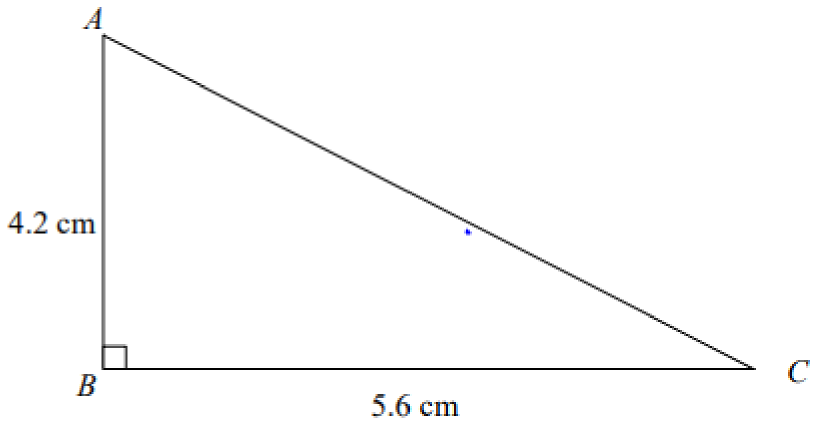
SECTION 5: PYTHAGORAS' THEOREM

You will be familiar with Pythagoras' theorem from GCSE Maths.
We can use Pythagoras' theorem to calculate the length of any of the sides on a right-angled triangle using the rule;

$$a^2 = b^2 + c^2$$

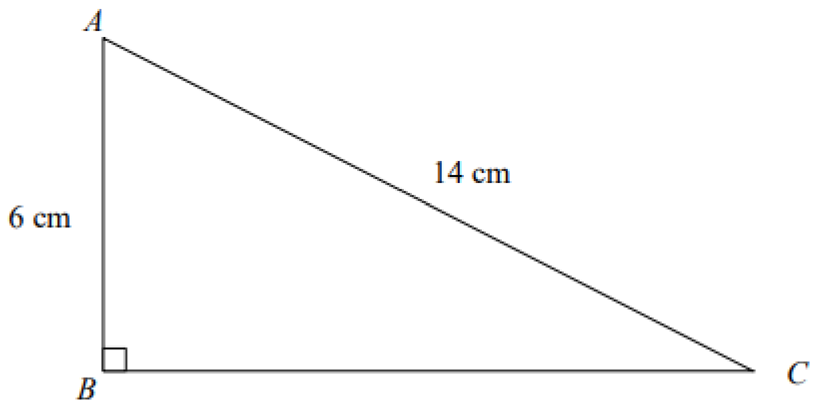


Example 1: Calculate the length of AC



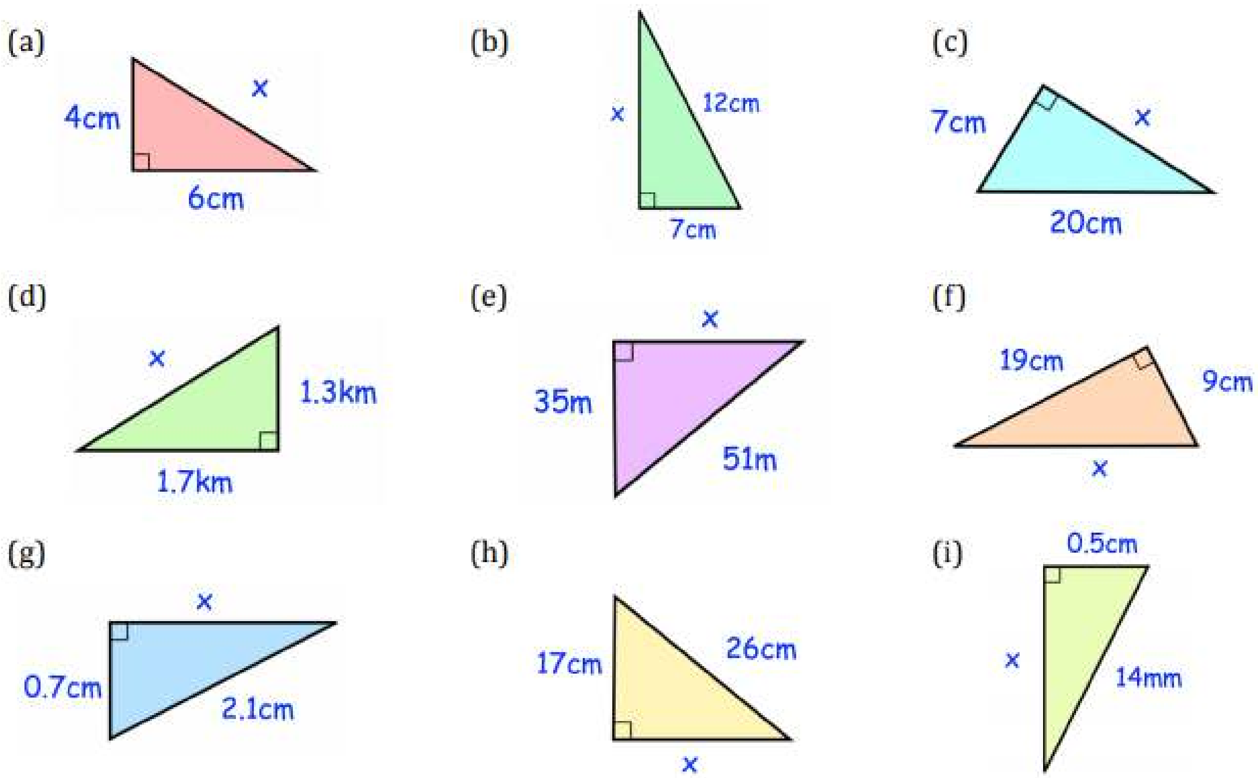
$AC^2 = 4.2^2 + 5.6^2$
$AC^2 = 17.64 + 31.36$
$AC^2 = 49$
$AC = \sqrt{49}$
$AC = 7$ cm

Example 2:   Calculate the length of BC, give your answer to 1dp



$BC^2 = 14^2 - 6^2$
$BC^2 = 196 - 36$
$BC^2 = 160$
$BC = \sqrt{160}$
$BC = 12.6$ cm

Exercise A



SECTION 6: STEM AND LEAF DIAGRAMS

Stem and leaf diagrams show data in a table format. They are a good way of organising data to find the averages and range. You should be able to construct and interpret stem and leaf diagrams. You should always remember to include/interpret the key.

Example 1: Here are the speeds, in miles per hour, of 16 cars:

31 52 43 49 36 35 33 29 54 43 44 46 42 39 55 48

Draw an ordered stem and leaf diagram for these speeds.

2	9
3	1, 3, 5, 6, 9
4	2, 3, 3, 4, 6, 8, 9
5	2, 4, 5

Key: 2 | 9 = 29mph

Example 2:

Anil counted the number of letters in each of 30 sentences in a newspaper.
Anil showed his results in a stem and leaf diagram.

0	8 8 9
1	1 2 3 4 4 8 9
2	0 3 5 5 7 7 8
3	2 2 3 3 6 6 8 8
4	1 2 3 3 5

Key 4 |1 stands for 41 letters

(a) Write down the number of sentences with 36 letters.

2, as 6 appears twice in the 30 row.

(b) Work out the range.

45 – 8 = 37

(c) Work out the median.

$$\frac{30+1}{2} = 15.5th\ value$$

15th value is 27 and 16th value is 27 so the median is 27.

EXERCISE

1) Draw ordered stem and leaf diagrams for the following data. Remember to include a key:

(a) 35, 50, 38, 44, 53, 41, 39, 45, 48, 55

(b) 18, 42, 5, 28, 33, 9, 15, 38, 32, 9, 11, 24, 40, 29, 24

(c) 153, 144, 148, 140, 149, 145, 144, 142, 158, 135, 140, 139, 160

(d) 3.4kg, 1.9kg, 2.8kg, 3.1kg, 5.1kg, 3.9kg, 4.8kg, 4.5kg, 2.2kg, 3.7kg,

2) The stem and leaf diagram below shows the ages of a group of people:

(a) How many people are there in the group?

(b) How old is the youngest member of the group?

(c) How old is the oldest member of the group?

(d) How many people are under 20?

(e) How many people are over 25?

Key: 1|4 means 14 years old

1	4 5 8
2	1 3 6 9 9
3	0 5 7
4	4

3) The stem and leaf diagram below shows heights of Mrs Smith's flowers:

(a) How many flowers does Mrs Smith have?

(b) What is the height of the shortest flower?

(c) What is the height of the tallest flower?

(d) How many flowers have a height of 14cm?

(e) How many flowers have a height greater than 40cm?

(f) What fraction of the flowers have a height under 20cm?

Key: 0|9 means 9cm

0	9
1	2 4 4 4 8 9
2	0 4 5 8
3	2 4 9
4	1 6 8 8
5	3

SECTION 7: HISTOGRAMS

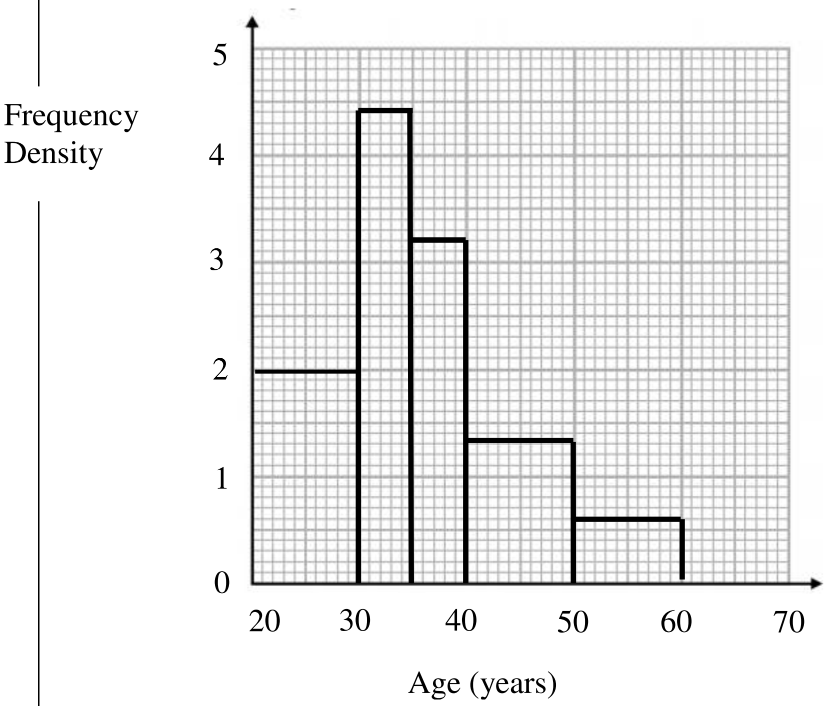
In a histogram the area of the bar represents the frequency of the data with the height representing the frequency density. Often the width of the classes(bars) are unequal. To calculate the frequency density we divide the frequency by the class width. You need to be able to construct and interpret histograms.

Example:

The table shows information about the age of 80 teachers.

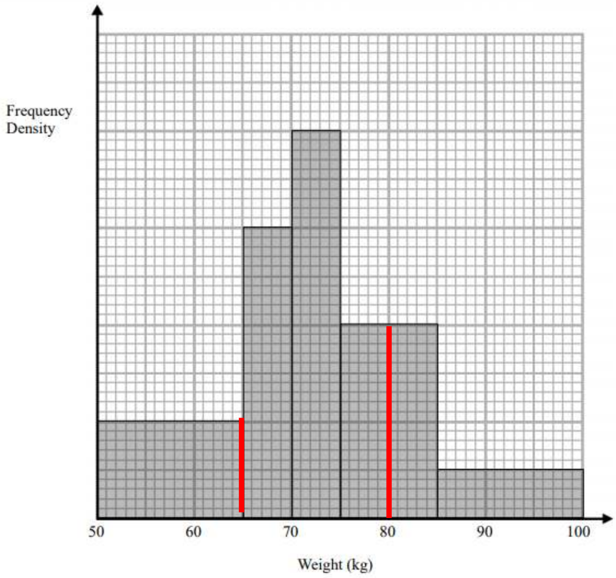
Age (years)	Frequency	Class width	Frequency density
$20 < a \leq 30$	20	10	2
$30 < a \leq 35$	22	5	4.4
$35 < a \leq 40$	16	5	3.2
$40 < a \leq 50$	13	10	1.3
$50 < a \leq 65$	9	15	0.6

We can then draw the histogram:



Example

The histogram shows information about the weight of pigs



30 pigs weigh between 50 and 65 kg.
Work out an estimate for the number of pigs which weigh more than 80kg.

150 squares = 30 pigs
25 squares = 5 pigs
175 = 35 pigs

Exercise A:

1) Draw a histogram for each set of data below:

(a)

Time, t seconds	Frequency
$0 \le t < 2$	10
$2 \le t < 4$	13
$4 \le t < 6$	18
$6 \le t < 10$	16
$10 \le t < 14$	8
$14 \le t < 20$	6

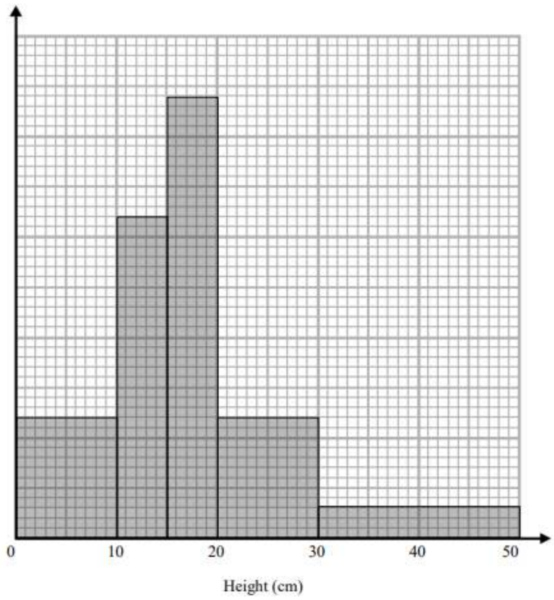
(b)

Length (cm)	Frequency
$0 \le L < 20$	10
$20 \le L < 30$	35
$30 \le L < 40$	65
$40 \le L < 80$	40

(c)

Mass, m kg	Frequency
$40 \le m < 50$	4
$50 \le m < 60$	7
$60 \le m < 70$	13
$70 \le m < 85$	12
$85 \le m < 100$	3
$100 \le m < 120$	3

2) The histogram shows information about the height of some plants.



Work out an estimate for the proportion of plants over 25cm tall

SOLUTIONS TO THE EXERCISES

SECTION 1:

Ex A

(a) 76.8ml (b) £417.60 (c) 85.8g

(d) 304.304 (e) 2238.75 (f) 8.4573kg

Ex B

Question 1: £4410

Question 2: 26620

Question 3: £9025

Question 4: £2105.74 or £2105.75

Question 5: 149.609 litres

Question 6: 419.4 cm or 4.194 m

SECTION 2

Ex A

(a) 9 (b) 40 © 4/4.8 (d) 20

(e) 8 (f) 100 (g) 40 (h) 16

Ex B

(a) 700 (b) 16000 © 14 (d) 20

(e) 6000 (f) 16000 (g) 400 (h) 5000

SECTION 3

Ex A

a) 16.5 b) 5.6 c) 0.48 d) £2.93 e) 2.64 f) 1.49

Ex B

a) 17.75 b) 54.4 c) 30.8 d) 139.8

Ex C

a) 20 b) 6 c) 2 d) 0 e) 8 f) 2

Ex D

a) $5 < t \leq 10$ b) $36 < t \leq 48$

SECTION 4

Ex A

(a) 3	(b) 4	(c) 4
(d) -1	(e) 2	(f) $-\frac{1}{4}$
(g) $-\frac{4}{5}$	(h) $-\frac{1}{3}$	(i) -1

Ex B

(a) $y = 3x - 1$	(b) $y = x + 6$	(c) $y = 4x + 12$
(d) $y = -x - 3$	(e) $y = 2x + 8$	(f) $y = -4x + 17$
(g) $y = -\frac{1}{5}x + 3$	(h) $y = -\frac{1}{2}x + 6\frac{1}{2}$	(i) $y = 3x + 25$

SECTION 5

Ex A

(a) 7.21cm	(b) 9.75 cm	(c) 18.73cm
(d) 2.14km	(e) 37.09m	(f) 21.02cm
(g) 1.98cm	(h) 19.67cm	(i) 13.08mm

SECTION 6

1)

(a)

3	5 8 9
4	1 4 5 8
5	0 3 5

Key: 4|1 means 41

(b)

0	5 9 9
1	1 5 8
2	4 4 8 9
3	2 3 8
4	0 2

Key 2|4 means 24

(c)

13	5 9
14	0 0 2 4 4 5 8 9
15	3 8
16	0

Key 13|5 means 135

(d)

1	9
2	2 8
3	1 4 7 9
4	5 8
5	1

Key 5|1 means 5.1kg

2)

(a) 12
(b) 14
(c) 44
(d) 3
(e) 7

3)

(a) 19
(b) 9cm
(c) 53cm
(d) 3
(e) 5

SECTION 7
Ex A
1) Histograms drawn 2) 3/17